ABSTRACT

Several diseases have affected the maize crop productivity, among them standing out Corn Stunt Spiroplasma (CSS) and Maize Bushy Stunt Phytoplasma (MBSP), which are respectively caused by *Spiroplasma kunkelii* and by phytoplasma. In Brazil the mollicutes are transmitted to maize plants by the leafhopper *Dalbulus maidis* which has a remarkable ability to travel long distances if the local climatic conditions are not favourable, e.g., low temperature or high pluviometric precipitation. This work presents a spatial-time deterministic model for analysing the dynamics of the pathosystem “mollicutes – leafhopper – maize” considering a macro region formed by several maize crop fields. From the simulated scenarios we imply that the prediction of the incidence of mollicutes in a maize crop field is only possible when all the other maize crop fields within the travel distance of the leafhopper is taken into consideration.

Keywords: Corn stunting diseases, Leafhopper, Mathematical model, Brazil

1. INTRODUCTION

The Brazilian corn production per year is more than 70 million tons, representing more than 40% of its total grain production (CONAB, 2013). There are two maize crop cultivation seasons in some regions of Brazil. At these regions, the corn harvest is usually in the summer, for the first maize crop cultivation, and in the autumn for the second maize crop cultivation season (named *safrinha*).

Several diseases have affected the maize crop productivity, among them standing out Corn Stunt Spiroplasma (CSS) and Maize Bushy Stunt Phytoplasma (MBSP), which are respectively caused by *Spiroplasma kunkelii* and by phytoplasma, both pathogens belonging to the class Mollicutes. In Brazil the mollicutes are transmitted to maize plants by the leafhopper *Dalbulus maidis* in a persistent propagative way (Nault, 1980; Oliveira et al. 2004). These diseases cause a reduction in plants growth, reddening or yellowing leaves, excessive proliferation of small spikes, and consequently, they reduce the maize crop productivity (Júnior, 2001; Oliveira et al. 2002). The importance of
stunting has increased in recent years, mainly due to the overlapping of crop cycles because maize plants are the main host of the leafhopper in Brazil. In addition, it has already been observed (Oliveira et al., 2002) that *D. maidis* has a remarkable ability to fly long distances if the local climatic conditions are not favourable, e.g., low temperature or high pluviometric precipitation (Oliveira et al. 2012).

Therefore, since *D. maidis* may be detected in uninfected fields in a surprising way due to its flying capability, the development of predictive mathematical models coupled with decision support systems becomes important for assisting the prevention and control of this insect vector.

In this study we present a spatial mathematical model for analysing the dynamics of the pathosystem “mollicutes – leafhopper – maize” considering a macro region formed by several maize crop fields. In a particular maize crop field the disease development was represented by a SEI compartmental model (Edelstein-Keshet, 2005) considering the infection cycle in the host (plant) and the insect vector infectivity (leafhopper). Following the idea of a traditional network flow model, spatially we consider a network of maize crop fields where each field represents a node. The spatial-time model proposed here is presented in details in section 2. The results obtained from computational simulation is described is section 3.

2. MATHEMATICAL MODEL

The local dynamics of MBSP and CSS is represented inside a crop field by a SEI compartmental model considering the infection cycle in the host (plant) and the insect vector infectivity (leafhopper). The plant compartments are susceptible (*S_h*) and infected (*I_h*) and analogously, we have *S_v* and *I_v* for the insect vector plus an exposed compartment (*E_v*) that represents the latency period (see Figure 1).

The model is given by the following system of ordinary differential equations:

\[
\begin{align*}
S_h & \quad \lambda_h \quad I_h \\
S_v & \quad \lambda_v \quad E_v \quad \frac{1}{\tau} \quad I_v \\
\end{align*}
\]

**Figure 1. Model diagram**
where $N_h = S_h + I_h$, $N_v = S_v + E_v + I_v$; $\tau$ is the latency period of the disease in the leafhopper; $\alpha_v$ and $\mu_v$ are the birth and death rates of the leafhopper, respectively; $\lambda_h$ and $\lambda_v$ are the infection forces for the population of hosts and insects, respectively, and $\phi_S$, $\phi_E$ and $\phi_I$ are insect migration fluxes.

Since the eggs deposited in infected plants will grow into infected leafhoppers, the incoming flux from nymphs will be the proportion of infected plants ($I_h/N_h$) multiplied by the number eggs deposited ($\alpha_v N_v$). An analogous expression applies for the susceptible compartment.

The infection forces are given by

$$\lambda_h = b p_h \frac{I_v}{N_v}, \quad \lambda_v = b p_v \frac{I_h}{N_h}$$

where $b$ is the probing rate of leafhoppers; $p_h$ is the probability of disease transmission from insects to plants and $p_v$ is the probability of disease acquisition by insects (see Table 1 for parameter values).

From the equations of the model, we have the dynamics of the total population of leafhoppers given by

$$\frac{dN_v}{dt} = (\alpha_v - \mu_v)N_v.$$

If we compare this expression with Verhulst equation, the mortality must satisfy

$$\mu_v = \frac{\alpha}{K} N_v,$$

where $K$ is the carrying capacity of leafhoppers.
The model considers a complex migration process of the leafhoppers across the network of maize crop fields, whose fluxes are represented by $\varphi_S$, $\varphi_E$ and $\varphi_I$ in the previous system of ODEs.

Although it is known that the wind plays a major role in the long distance travel of the leafhopper, the whole mechanism is not properly known (Oliveira and Lopes, 2004). However we simplify the whole process considering that the leafhopper will emigrate from an area whose leafhopper population is higher than its carrying capacity (an area with high emigration potential) to an area with a lower emigration potential, i.e., the leafhopper population is lower than the local carrying capacity at the present moment.

The concept of *emigration potential* is introduced in this work to quantify how willing an insect vector which is living in a specific field is for emigrating to another one. This value is defined by the difference between the insect vector total population and the maize crop field's carrying capacity, which is considered as a function of the dynamic local climate conditions:

$$ P(t) = N_v(t) - K(t); $$

where $N_v$ is the total vector population in the crop field, i.e., $N_v = S_v(t) + I_v(t)$, and $K(t)$ is the carrying capacity of the field.

The emigration flux between two fields will happen at a time $t$ if the emigration potential at the source field is higher than the destination and if the destination field is the one with the minimum emigration potential among the neighbors of the source crop field.

The flux functions are constructed in order to speed the diminishing of the population towards the carrying capacity in a Verhulst sense:

$$ \varphi_S + \varphi_E + \varphi_I = r N_v \left(1 - \frac{N_v}{K}\right), $$

where we used the usual notation for the negative part of a function:

$$ f^- (x) = \max(-f(x), 0). $$

The decaying rate $r$ is defined as being proportional to the difference of potentials of these two crop fields ($\Delta P$) and the distance between them ($d$) will be also used as a logistic decaying in order to model the travel capacity of the leafhoppers, known to at least of 20km (Oliveira et al, 2011):
where we choose $\beta_1\beta_2 = 20$ in order to get $r(20) = 0.5$ (see Figure 2) and $\beta_3 = 0.02$.

We can finally define the fluxes as

$$
\phi_s(t; d, \Delta P) = \frac{\beta_3 \Delta P}{1 + e^{\frac{-d}{10}}} \left[1 - \frac{N_p}{K}\right]^{-} S_p,
$$

$$
\phi_e(t; d, \Delta P) = \frac{\beta_3 \Delta P}{1 + e^{\frac{-d}{10}}} \left[1 - \frac{N_p}{K}\right]^{-} E_p,
$$

$$
\phi_l(t; d, \Delta P) = \frac{\beta_3 \Delta P}{1 + e^{\frac{-d}{10}}} \left[1 - \frac{N_p}{K}\right]^{-} I_p.
$$

Each emigration flux is computed independently and the final migration flux is computed for each crop field as the sum of the incoming migration minus the emigration calculated by the above formulas.

Figure 2. Logistic equation for distance damping in the decaying rate $r$

3. RESULTS

The first simulation was performed using the parameters value shown on Table 1 for only one maize crop field (see Figure 3a), i.e., with the fluxes not contributing for the dynamics. In Figure 3b we have the total population of leafhoppers altogether with the carrying capacity. A second maize crop field (see Figures 3c and 3d) was also simulated using the parameters of Table 1 but with different initial conditions for the system.
of ODEs and a dislocated carrying capacity function from what was used in the first field. As expected for both fields, $N_v$ approaches the carrying capacity $K$ because of the Verhulst formulation of the death rate of the leafhoppers.

On Figures 4a and 4c we have included the fluxes in the dynamics of the disease on the first and second fields, respectively. But the distance between them was defined as being much larger than 20km, meaning that an emigrating flux coming out from the fields exist but there is not an incoming component on this. Comparing 3b to 4a we have a faster decaying of $N_v$ in the direction of $K$ when $N_v > K$ is. However the dynamics is as slow as before when $N_v < K$. And comparing 3d with 4c there is not any difference, since $N_v < K$ all the time.

Figure 3: The dynamics of mollicutes is simulated for two maize crop fields using the parameters values shown on Table 1. The migration fluxes are not taken into account.
Figure 4: The total population of vectors \( N_v \) in a similar scenario from the simulated on Figure 3 but taking the migration fluxes into account. In Figures 4a and 4c the first and second crop fields are placed very far from each other implying only emigration fluxes. In Figures 4b and 4d the fields are placed only 10km apart.

Placing the two fields at only 10km away, the total population of leafhoppers on the first and second fields are shown on Figures 4b and 4d, respectively. Now we see a dramatic change in the dynamics of the total population of leafhopper, which is now much closer to the local carrying capacity of each field since there is an intense flow between the two maize crop fields.

4. CONCLUSIONS AND CONSIDERATIONS

The model proposed in this work was developed in order to replicate the long distance traveled by leafhoppers leaving maize crop fields when the climactic conditions are not favorable. Although the complex process of wind transport is not fully understood and could not be modeled explicitly, a network flux process was proposed in order to retain the most important features of the natural flow process.

From the model proposed it is clear that it would not be possible to predict the dynamics of the leafhopper population on a maize crop field without analyzing the neighboring fields as well. But a practical use for such a prediction tool would be possible only with the determination of the parameters of the model by means of experimental results on a large area composed by a group of crop fields closer than the maximum travelling distance of the leafhopper.

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6. REFERENCES


